

## Integrated Pathway: Mathematics II

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1=0$  to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

**Critical Area 2:** Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

**Critical Area 3:** Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

**Critical Area 4:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

**Critical Area 5:** Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

**Critical Area 6:** In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

| Units   | Includes Standard Clusters*   | Mathematical Practice Standards  |
|---|---|--|
| <b>Unit 1</b><br>Extending the Number System                        | <ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> <li>Use properties of rational and irrational numbers.</li> <li>Perform arithmetic operations with complex numbers.</li> <li>Perform arithmetic operations on polynomials.</li> </ul>   |  |
| <b>Unit 2</b><br>Quadratic Functions and Modeling                   | <ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> | <p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p>  |
| <b>Unit 3†</b><br>Expressions and Equations                         | <ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Create equations that describe numbers or relationships.</li> <li>Solve equations and inequalities in one variable.</li> <li>Use complex numbers in polynomial identities and equations.</li> <li>Solve systems of equations.</li> </ul>                  | <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> |
| <b>Unit 4</b><br>Applications of Probability                        | <ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>   | <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>   |
| <b>Unit 5</b><br>Similarity, Right Triangle Trigonometry, and Proof | <ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> <li>Prove geometric theorems.</li> <li>Prove theorems involving similarity.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Define trigonometric ratios and solve problems involving right triangles.</li> <li>Prove and apply trigonometric identities.</li> </ul>   |  |
| <b>Unit 6</b><br>Circles With and Without Coordinates               | <ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> <li>Find arc lengths and areas of sectors of circles.</li> <li>Translate between the geometric description and the equation for a conic section.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Explain volume formulas and use them to solve problems.</li> </ul>                    |  |

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.

## Unit 1: Extending the Number System

Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 2, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1=0$  to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

| Unit 1: Extending the Number System  |  |
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| Clusters with Instructional Notes  | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> </ul>  | <p>N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3)3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i></p> <p>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p> |
| <ul style="list-style-type: none"> <li>Use properties of rational and irrational numbers.</li> </ul> <p><i>Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.</i></p>                                     | <p>N.RN.3 Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.</p>  |
| <ul style="list-style-type: none"> <li>Perform arithmetic operations with complex numbers.</li> </ul> <p><i>Limit to multiplications that involve <math>i^2</math> as the highest power of <math>i</math>.</i></p>                               | <p>N.CN.1 Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p> <p>N.CN.2 Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p>   |
| <ul style="list-style-type: none"> <li>Perform arithmetic operations on polynomials.</li> </ul> <p><i>Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of <math>x</math>.</i></p> | <p>A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>  |

## Unit 2: Quadratic Functions and Modeling

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

| Unit 2: Quadratic Functions and Modeling  |   |
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| Clusters with Instructional Notes   | Common Core State Standards   |
| <ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.</i></p>   | <p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>★</p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>★</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★</p>   |
| <ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Mathematics I on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.</i></p> <p><i>Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.</i></p> | <p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> </ol> <p>F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <ol style="list-style-type: none"> <li>Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</li> <li>Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i></li> </ol> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p> |
| <ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Focus on situations that exhibit a quadratic or exponential relationship.</i></p>   | <p>F.BF.1 Write a function that describes a relationship between two quantities.★</p> <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ol>  |

| Unit 2: Quadratic Functions and Modeling   |  |
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| Clusters with Instructional Notes  | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>For F.BF.3, focus on quadratic functions and consider including absolute value functions.. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as <math>f(x) = x^2</math>, <math>x &gt; 0</math>.</i></p> | <p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p> <p>F.BF.4 Find inverse functions.</p> <p>a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></p> |
| <ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>Compare linear and exponential growth studied in Mathematics I to quadratic growth.</i></p>  | <p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>   |

## Unit 3: Expressions and Equations

Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

| Unit 3: Expressions and Equations   |  |
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| Clusters with Instructional Notes   | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.</i></p>   | <p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol> <p>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i></p> |
| <ul style="list-style-type: none"> <li>Write expressions in equivalent forms to solve problems.</li> </ul> <p><i>It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.</i></p> | <p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*</p> <ol style="list-style-type: none"> <li>Factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</li> <li>Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></li> </ol>             |
| <ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Extend work on linear and exponential equations in Mathematics I to quadratic equations. Extend A.CED.4 to formulas involving squared variables.</i></p>   | <p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>  |
| <ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Extend to solving any quadratic equation with real coefficients, including those with complex solutions.</i></p>  | <p>A.REI.4 Solve quadratic equations in one variable.</p> <ol style="list-style-type: none"> <li>Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</li> <li>Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</li> </ol>                  |
| <ul style="list-style-type: none"> <li>Use complex numbers in polynomial identities and equations.</li> </ul> <p><i>Limit to quadratics with real coefficients.</i></p>   | <p>N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.</p> <p>N.CN.8 (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i></p> <p>N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>   |

| Unit 3: Expressions and Equations  |  |
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| Clusters with Instructional Notes  | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between <math>x^2 + y^2 = 1</math> and <math>y = (x+1)/2</math> leads to the point <math>(3/5, 4/5)</math> on the unit circle, corresponding to the Pythagorean triple <math>3^2 + 4^2 = 5^2</math>.</i></p> | <p>A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p> |

## Unit 4: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

| Unit 4: Applications of Probability  |  |
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| Clusters and Instructional Notes   | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> </ul> <p><i>Build on work with two-way tables from Mathematics I Unit 4 (S.ID.5) to develop understanding of conditional probability and independence.</i></p>   | <p>S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p> <p>S.CP.2 Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>S.CP.3 Understand the conditional probability of <math>A</math> given <math>B</math> as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of <math>A</math> and <math>B</math> as saying that the conditional probability of <math>A</math> given <math>B</math> is the same as the probability of <math>A</math>, and the conditional probability of <math>B</math> given <math>A</math> is the same as the probability of <math>B</math>.</p> <p>S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p>S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p> |
| <ul style="list-style-type: none"> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> </ul>   | <p>S.CP.6 Find the conditional probability of <math>A</math> given <math>B</math> as the fraction of <math>B</math>'s outcomes that also belong to <math>A</math>, and interpret the answer in terms of the model.</p> <p>S.CP.7 Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model.</p> <p>S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model.</p> <p>S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.</p>   |
| <ul style="list-style-type: none"> <li>Use probability to evaluate outcomes of decisions.</li> </ul> <p><i>This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.</i></p> | <p>S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p> <p>S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>   |



## Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.

It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

| Unit 5: Similarity, Right Triangle Trigonometry, and Proof  |  |
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| Clusters and Instructional Notes  | Common Core State Standards  |
| <ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> </ul>   | <p>G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.</p> <ol style="list-style-type: none"> <li>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</li> <li>The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</li> </ol> <p>G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p> <p>G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>  |
| <ul style="list-style-type: none"> <li>Prove geometric theorems.</li> </ul> <p><i>Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 6.</i></p> | <p>G.CO.9 Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p> <p>G.CO.10 Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to <math>180^\circ</math>; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p> <p>G.CO.11 Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i></p> |
| <ul style="list-style-type: none"> <li>Prove theorems involving similarity.</li> </ul>  | <p>G.SRT.4 Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i></p> <p>G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</p>   |
| <ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul>   | <p>G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</p>  |
| <ul style="list-style-type: none"> <li>Define trigonometric ratios and solve problems involving right triangles.</li> </ul>   | <p>G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p> <p>G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.</p> <p>G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p>   |

| Unit 5: Similarity, Right Triangle Trigonometry, and Proof  |   |
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| Clusters and Instructional Notes  | Common Core State Standards   |
| <ul style="list-style-type: none"> <li>Prove and apply trigonometric identities.</li> </ul> <p><i>In this course, limit <math>\theta</math> to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. A course with a greater focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could continue to be limited to acute angles in Mathematics II.</i></p> <p><i>Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.</i></p> | <p>F.TF.8 Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math>, given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math>, and the quadrant of the angle.</p> |

## Unit 6: Circles With and Without Coordinates

In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

| Unit 6: Circles With and Without Coordinates   |   |
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| Clusters and Instructional Notes   | Common Core State Standards   |
| <ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> </ul>   | <p>G.C.1 Prove that all circles are similar.</p> <p>G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i></p> <p>G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</p> <p>G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.</p> |
| <ul style="list-style-type: none"> <li>Find arc lengths and areas of sectors of circles.</li> </ul> <p><i>Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.</i></p>  | <p>G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p>   |
| <ul style="list-style-type: none"> <li>Translate between the geometric description and the equation for a conic section.</li> </ul> <p><i>Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.</i></p>  | <p>G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</p> <p>G.GPE.2 Derive the equation of a parabola given a focus and directrix.</p>  |
| <ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul> <p><i>Include simple proofs involving circles.</i></p>   | <p>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p>   |
| <ul style="list-style-type: none"> <li>Explain volume formulas and use them to solve problems.</li> </ul> <p><i>Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor <math>k</math>, its area is <math>k^2</math> times the area of the first. Similarly, volumes of solid figures scale by <math>k^3</math> under a similarity transformation with scale factor <math>k</math>.</i></p> | <p>G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i></p> <p>G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*</p>  |